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Question Paper Code : X 20784

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fourth Semester
Computer Science and Engineering
MA 6453 - PROBABILITY AND QUEUEING THEORY
(Common to Mechanical Engineering (Sandwich) and Information Technology)
(Regulations 2013)

Time : Three Hours — Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. A continuous random variable X has the probability density function given by $f(x)\begin{cases}\lambda(1+x^2), & 1 \le x \le 5\\ 0, & \text{otherwise}\end{cases}$. Find λ and P (X< 4).
2. What is meant by memoryless property ? Which discrete distribution follows this property ?
3. The joint pdf of a two-dimensional random variable (X, Y) is given by $f(x,y)=\begin{cases}kxe^{-y}, & 0 \le x \le 2, y > 0\\ 0, & \text{otherwise}\end{cases}$. Find the value of 'k'.
4. In a partially destroyed laboratory, record of an analysis of correlation data, the following results only are legible : Variance of X = 9; Regression equations are 8X – 10Y + 66 = 0 and 40X – 18Y = 214. What are the mean values of X and Y ?
5. The random process X(t) is given by X(t) = Y cos(2πt), t > 0, where Y is a R.V. with E(Y) = 1. Is the process X(t) stationary ?
6. Derive the autocorrelation function for a Poisson process with rate λ.
7. State the basic characteristic of queueing system.
8. Write the Little's formula for queueing system.
9. State Pollaczek - Khintchine formula for (M/G/1) queuing model.
10. Write down the traffic equation for open Jackson network.

PART – B **(5×16=80 Marks)**

11. a) i) The CDF of the random variable of X is given by

$$F_x(x) = \begin{cases} 0; x < 0 \\ x + \frac{1}{2}; 0 \le x \le \frac{1}{2} \\ 1; x > \frac{1}{2} \end{cases}.$$

Draw the graph of the CDF. Compute $P(X > 1/4), \left(\frac{1}{3} < X \le \frac{1}{2}\right)$. **(8)**

ii) Find the moment generating function of a geometrically distributed random variable and hence find the mean and variance. **(8)**

(OR)

b) i) Messages arrive at a switch board in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events:

1) exactly two messages arrive within one hour
2) no message arrives within one hour
3) at least three messages arrive within one hour. **(8)**

ii) The peak temperature T, as measured in degrees Fahrenheit , on a particular day is the Gaussian (85, 10) random variable. What is P(T > 100). P(T < 60) and P(70 ≤ T ≤ 100) ? **(8)**

12. a) i) Find the constant k such that

$$f(x,y) = \begin{cases} k(x+1)e^{-y}, & 0 < x < 1, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

is a joint p.d.f. of the continuous R.V. (X, Y). Are X and Y independent R.Vs ? Explain. **(8)**

ii) The joint p.d.f. of the continuous R.V. (X, Y) is given as

$$f(x,y) = \begin{cases} e^{-(x+y)}, & x > 0, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the p.d.f. of the $R.V. U = \frac{X}{Y}$. **(8)**

(OR)

b) i) Let the joint p.d.f. of R.V. (X, Y) be given as

$$f(x,y)=\begin{cases} Cxy^2, & 0 \le x \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

. Determine (1) the value of C (2) the marginal p.d.fs of X and Y (3) the conditional p.d.f. f(x/y) of X given Y= y. **(8)**

ii) A joint probability mass function of the discrete R.Vs X and Y is given as

$$P(X=x, Y=y)=\begin{cases} \dfrac{x+y}{32}, & x=1,2, y=1,2,3,4 \\ 0, & \text{otherwise} \end{cases}$$

Compute the covariance of X and Y. **(8)**

13\. a) i) Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes (1) exactly 4 customers arrive (2) greater than 4 customers arrive (3) fewer than 4 customers arrive. **(8)**

ii) The process $\{X(t)\}$ whose probability distribution under certain condition is given by

$$P[X(t)=n]=\begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\ldots. \\ \dfrac{at}{1+at}, & n=0 \end{cases}$$

Show that $\{X(t)\}$ is not stationary. **(8)**

(OR)

b) i) A man either drives a car (or) catches a train to go to office each day. He never goes two days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared.

Find :

1) the probability that he takes a train on the third day

2) the probability that he drives to work in the long run. **(8)**

ii) Show that the random process $\{X(t) = A\cos(\omega_o t + \theta)\}$ is wide-sense stationary, if A and ω_0, are constants and θ is uniformly distributed random variable in $(0, 2\pi)$. **(8)**

14. a) Derive the steady-state probabilities of the number of customers in M/M/1 queueing system from the birth and death processes and hence deduce that the average measures such as expected system size L_s. expected queue size L_q, expected waiting time in system W_s and expected waiting time in queue W_q. **(16)**

(OR)

b) A petrol pump station has 4 pumps. The service time follows the exponential distribution with a mean of 6 min. and cars arrive for service in a Poisson process at the rate of 30 cars per hour. What is the probability that an arrival would have to wait in line ? Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system. **(16)**

15. a) In a network of 3 service stations 1,2,3 customers arrive at 1,2.3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective rates 10, 50, 100. A customer completing service at station 1 is equally likely to go to station 2 or go to station 3 or leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3. 1 is equally likely to go to station 2 or leave the system. What is the average number of customers in the system ? And what is the average time a customer spends in the system ? **(16)**

(OR)

b) Consider a queuing system where arrivals are according to a Poisson distribution with mean 5 per Hour. Find the expected waiting time in the system. If the service time distribution is

i) Uniform between: t = 5 minutes and t = 15 minutes

ii) Normal with mean 3 minutes and standard deviation 2 minutes. **(16)**
